QJEGH
Research article
Quarterly Journal of Engineering Geology and Hydrogeology
Published Online First
https://doi.org/10.1144/qjegh2020-093

Geological influence on the index properties variability and shear strength probability density functions

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Abstract: Determining soil properties variability in geotechnical engineering is one of the most important tasks in reliability-based designs (RBDs). However, these analyses have been carried without taking into account the influence of the geological origin on the different aspects that alter the soil properties variability. Therefore, two types of geological formations are analysed: residual soils (stationary origin) and mudflows (dynamic origin). First, the index properties variability was evaluated for each geology, where mudflows are less variable in comparison with the residual soils. It was confirmed that the correlations of the effective friction angle should not be used for high-plasticity and fine-grained soils; however, the shape characteristics of the probability density functions (PDFs) of both effective and total parameters depend on the geological origin. The undrained compressive strength (q_u) analyses show that geology influences the shape characteristics of the PDF and is directly proportional to the $(N_1)_{60}$ PDF. From the results, mudflows have a q_u PDF with a lognormal tendency, which is inferred to be due to the possible presence of rock fragments and randomness related to the soil's formation. However, the residual soils, under the same state of weathering, tend to have a normal q_u PDF, possibly owing to the stationary origin of these soils.

Supplementary material: Datasets are available at https://doi.org/10.6084/m9.figshare.c.5420240

Received 12 May 2020; **revised** 28 April 2021; **accepted** 7 May 2021

Geotechnical engineering requires various laboratory and field tests to determine the different soil properties for designing foundations and retaining walls, and for analysing slope stability. However, owing to the high inherent variability of geotechnical properties, soil is one of the most difficult materials to characterize realistically (Fenton and Griffiths 2008). Therefore, an evaluation of geotechnical variability will facilitate the use of reliability-based designs (RBDs) in geotechnical engineering to reduce the inherent uncertainties of soil behaviour. RBDs require the prior definition of geotechnical properties' probability density functions (PDFs), which is not usually performed owing to limited field and laboratory research. However, the Bayesian method has been recently adapted to obtain the PDF from limited field or laboratory data (e.g. Wang and Cao 2013; Wang and Aladejare 2015; Wang *et al.* 2016*a*, *b*). Regardless, the geological influence on the PDF shape is missing.

According to Hamedifar *et al.* (2014) and Wang *et al.* (2016*a*, *b*), the uncertainties of the index properties' (e.g. Atterberg limits and grain size distribution) overall geotechnical identification and the shear strength properties have been classified as inherent uncertainty, measurement errors, modelling uncertainty, variability in human and organizational task performance as well as gaps in knowledge development and utilization. A number of papers have evaluated these uncertainties through various statistical analyses (e.g. Lumb 1966, 1970; Phoon *et al.* 1995; Phoon and Kulhawy 1999*a*, *b*; Uzielli *et al.* 2007; Wang *et al.* 2016*a*, *b*). The influence of the geological origin has usually been considered in the Bayesian method as prior knowledge in geotechnical characterization (Cao *et al.* 2016). Geological origin consists of a categorical classification of the processes of weathering, erosion and transportation to explain the most relevant aspects of the soil structure. However, the influence of geological origin on the variability of geotechnical properties has been missing in geotechnical engineering RBDs.

This paper presents a statistical analysis to evaluate the influence of geological origin on soil variability to achieve a better RBD for similar geological conditions. To achieve this goal, we analysed two soils with different geological origins to understand how the processes of geological formation influence the variability of soil parameters. We then analysed two types of geological formations. The first is characterized as stationary soil represented by residual soils (soil formed from the weathering of the *in situ* rock), and the second is formed by dynamic processes known as mudflows (soil formed from ancient landslides).

We carried out soil variability analyses using geotechnical tests collected and reported in the literature in the city of Medellin. A geologist previously identified the geological origin of all the recovered samples for the data analyses. The same laboratory and equipment were used to perform the laboratory tests and standard penetration test (SPT) to prevent uncertainties such as measurement errors and variation in organizational task performance in the geological statistical analyses. The SPT was available from the collected geotechnical tests. Therefore, we evaluated the SPT's capability to obtain different geotechnical parameters according to the geological context.

The SPT remains one of the most used field tests in geotechnical engineering to obtain shear strength properties, owing to the difficulty in obtaining high-quality undisturbed samples, the high costs of other field tests and the simplicity of the SPT equipment (e.g. Sivrikaya and Toğrol 2006). The SPT is performed for soil sample recovery, identification of geological layers, N value determination and, subsequently, for the estimation of geotechnical shear strength properties through a variety of N value correlations of the peak effective friction angle (ϕ') and the undrained compressive strength (q_u). Although the SPT test was originally developed for coarse-grained soils, it can be applied to any type of soil, including fine-grained soils. However, the applicability of SPT–N correlations to geotechnical designs is still highly debated (e.g. Décourt 1990; Sivrikaya and Toğrol 2006; Mendes and Lorandi 2008; Hettiarachchi and Brown 2009; Nassaji and Kalantari 2011; Viviescas *et al.* 2019).

SPT correlations, which are empirical in nature, were developed under particular conditions and without a statistical analysis or regression analysis. Therefore, these correlations cannot be considered particularly accurate in a few cases because the SPT is not completely standardized (Clayton 1995). The use of these correlations in determining geotechnical parameters can involve significant uncertainties, especially when the SPT has limitations owing to the susceptibility of the results to factors such as the presence of rock fragments, variations in the state of weathering, drilling methods, borehole sizes, energy corrections and human errors. Therefore, the uncertainties of the SPT's correlated parameters may lead to possible high-risk designs owing to the susceptibility of the N value to overvalued properties. Given this flaw, we evaluated the most appropriate q_u–N correlation according to the geological origin as a method to minimize uncertainty.

The main objective of this paper is to evaluate how the geological origin of soil influences the variability of the geotechnical parameters. Therefore, the paper evaluates the overall data found in different projects in the same geological unit to assess the geological influence of geotechnical properties on a larger scale. As was shown by Baynes (2010), the relationship between engineering geology and geotechnical engineering provides a powerful argument to decrease the geotechnical risks that arise from inadequate understanding of the geological component of the soil conditions. Therefore, taking into account the influence of the geological origin on the soil variability will allow us to reduce, to some extent, the uncertainty between the designs and the field behaviour of geostructures (e.g. Fookes 1997; Viviescas *et al.* 2017).

Common statistical concepts for modelling geotechnical parameters

According to Griffiths *et al.* (2002), the soil is a complex engineering material that is difficult to characterize realistically, with parameters that can vary greatly from site to site. Therefore, various statistical parameters, such as the mean and standard deviation, are normally used to identify the geotechnical variability, as reported in the studies carried out by Lacasse and Nadim (1998), Phoon and Kulhawy (1999*a*, *b*), Baecher and Christian (2003) and Akbas and Kulhawy (2010). However, researchers have commonly analysed soil variability using the coefficient of variation (Cv), which is defined as the standard deviation (σ) divided by the mean (μ) as follows (e.g. Lee *et al.* 1983; Phoon and Kulhawy 1999*a*; Hicks and Samy 2002; Uzielli *et al.* 2007):

$$\mathrm{Cv} = \frac{\sigma}{\mu}. \tag{1}$$

When there is insufficient information to characterize the soil properties' variability, a Cv obtained from the literature is used in geotechnical engineering because the Cv shows low temporal and spatial sensitivity (Phoon and Ching 2013). According to Lee *et al.* (1983) and Uzielli *et al.* (2007), the Cv varies from 6 to 46% for the natural water content, 6 to 39% for the liquid limit, 6 to 34% for the plastic limit, 9 to 57% for the plasticity index, 6 to 80% for the undrained shear strength (c_u) and 4 to 15% for the peak friction angle.

The previous Cv ranges were estimated on soils with similar grain size distributions and were not determined according to the geological origin of the materials. Similar ranges can be expected in soils with a similar geological origin and grain size distributions. However, the use of generic guidelines suggested in the literature can be unconservative (Akbas and Kulhawy 2010). Nonetheless, RBD requires the definition of an accurate PDF of each variable to run the mathematical models to define the state of risk of geostructures.

The various statistical tests used in the paper are defined formally in the Appendix.

Probability density functions (PDFs)

The PDF of a random variable is a function that assigns the probability of occurrence to each event defined in the histogram of frequencies. The PDF used in the RBD identifies the probabilities of occurrences of the soil properties that are susceptible to important changes and whose behaviour can be described only by inductive statistical analysis (Baecher and Christian 2003).

The random processes in soil are usually represented by a normal or lognormal PDF (e.g. Baecher and Christian 2003; Cherubini *et al.* 2006; Sivakumar and Srivastava 2007; Uzielli *et al.* 2007; Fenton and Griffiths 2008; Papaioannou and Straub 2012; Fan *et al.* 2013; Wu 2013). These functions are widely used for literature when it is not possible to obtain a specific site's PDF. However, an erroneous PDF implementation can generate RBDs with inaccurate probabilities of failure as a result of simplification effects. Therefore, we will evaluate various PDFs according to the hypothesis tests to define the most accurate PDF.

Researchers have extensively studied the accuracy with which these functions describe the variability of the soil properties, corroborating that a PDF is a natural and inherent characteristic of the soil (Phoon 2008). In other cases, under specific conditions, other functions can be used to adjust the sampling distributions more accurately. These PDFs are defined for soils without considering the geological origin and are usually obtained through laboratory tests.

Characteristics of the analysed geology

We evaluated the geological influence in geotechnical properties' variability by using soil with abrupt changes and within a stationary environment. We used El Poblado mudflows and San Diego Stock residual soil to represent the mudflows and residual soils scrutinized in this study, respectively. All geological formations are located in the Antioquia department in the city of Medellin. However, these soils are commonly found in tropical and high mountain geological environments. The locations of the geological and the analysed projects are presented in Figure 1.

Mudflows

Mudflow soils generally comprise at least 50% silt or clay and 30% water (Wicander and Monroe 1999). In particular, the mudflows from El Poblado are formed by blocks of varying sizes, with a moderate to high degree of weathering (AMVA 2006). These soils were formed by previous landslides that were subjected to transportation and particle sorting, which can lead to tremendous uncertainties in the geotechnical shear strength properties owing to the void ratio variation of the location of the deposit (Zhao *et al.* 2013; Zhao and Zhang 2014). Figure 2 is a photograph of the El Poblado mudflow soil sample.

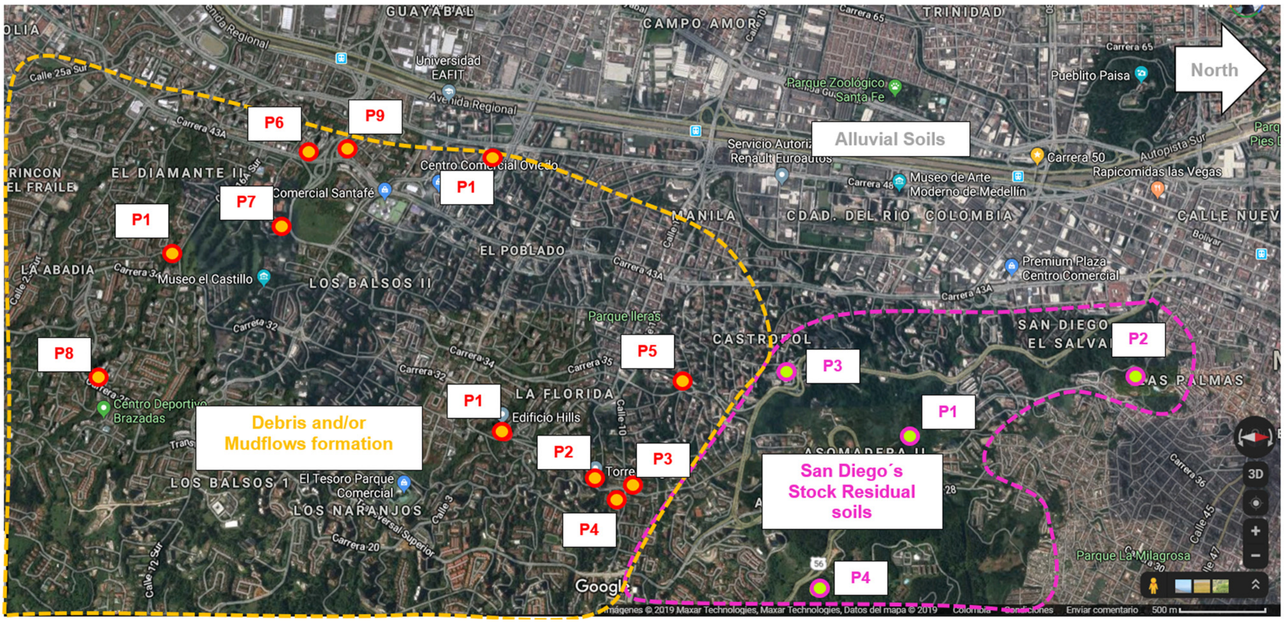


Fig. 1. Location of the analysed projects and local studied geologies in Medellin, Colombia (taken from Google Maps and adapted from AMVA (2006)).

Fig. 2. Poblado mudflow sample photograph.

Fig. 3. San Diego IC residual soil sample photograph.

Residual soils

Residual soils are materials that form directly from the weathering of the *in situ* rocks. Deere and Patton (1971) described a typical residual soil profile that depends on the state of weathering. They divided the profile into three main horizons, (I) residual soils, (II) weathered rock and (III) unweathered rock, which they subsequently divided into another three categories, A to C, A being the more weathered soil and C the less weathered of the profile.

The San Diego Stock, according to Restrepo and Toussaint (1984), is a plutonic body with basic igneous rocks, with a composition that varies from diorite to gabbro. The main feature of this geological unit is that it exhibits an advanced weathering process favoured by the climatic and topographical conditions, developing residual soils that can be up to 45 m in thickness. The collected soils are classified as a residual soil in a C state of weathering, also known as an IC state of weathering according to the Deere and Patton (1971) classification system. San Diego Stock residual soils are mainly composed of silt, with clay often changing to fine yellow and white sand with brown spots, as shown in Figure 3.

Results for index properties

The laboratory tests conducted on the soil samples recovered at the two distinct geological formations included water content (ASTM D2216 2010), Atterberg limits (ASTM D4318 2014), grain size distribution (ASTM-D422 2007) and unit weight (ASTM D7263 2009). To prevent measurement uncertainties in the statistical analyses, all the laboratory tests were conducted in the same laboratory and with the same equipment. According to the previous results, the soils were classified according to the Unified Soil Classification System (USCS). Table 1 presents a summary of the information collected at each location, including the number of projects, number of boreholes, $(N_1)_{60}$ and the USCS. Figure 4 shows the overall index properties results in a Casagrande chart for both geological soils.

Mudflow results

According to Figure 5, a total of 28 samples for El Poblado's mudflows have a predominant classification as MH (silt), surpassing the samples in other classifications by far. Therefore, the results

Table 1. *Numbers of data obtained from each geology unit for the index properties characterization*

Location, geology	Number of data					Maximum depth (m)	Analysed area (km^2)
	Projects	Borings	$(N_1)_{60}$	γ_m	USCS		
Poblado, mudflows	11	30	251	15	33	19.5	3
San Diego, IC residual	4	22	140	5	29	10.5	1.1

γ_m, moist unit weight; USCS, Unified Soil Classification System.

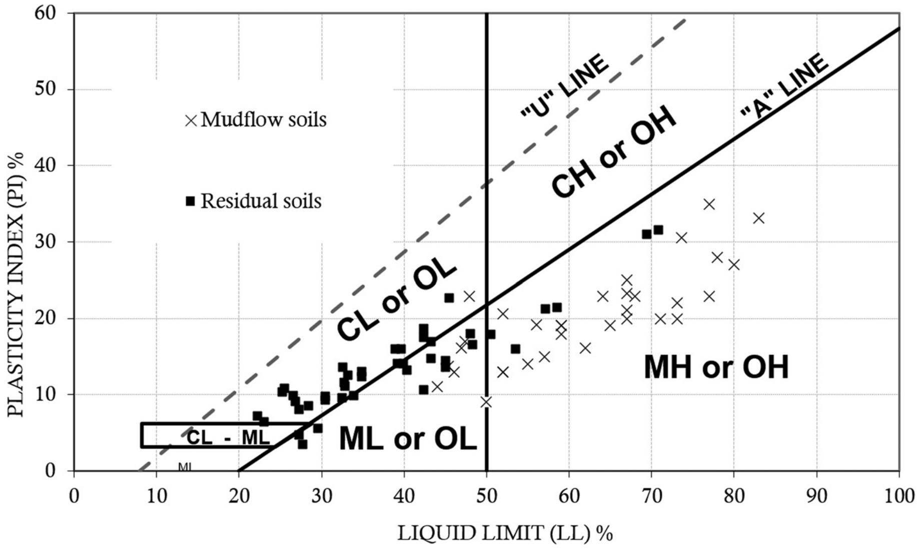


Fig. 4. Casagrande plasticity chart with the index properties result for mudflow and residual soils.

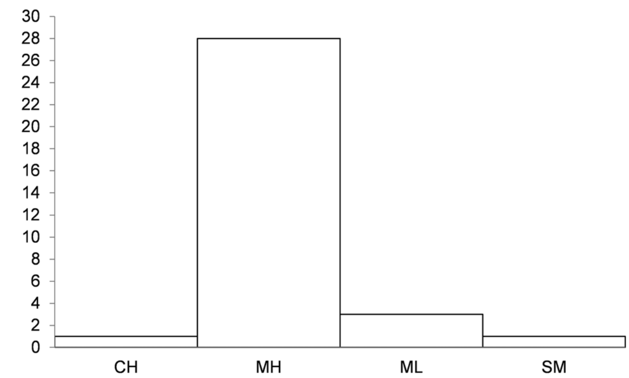


Fig. 5. Frequency histogram of the USCS for the Poblado mudflows (taken from Viviescas and Osorio 2015).

showed that mudflows are mainly classified as MH in the area of analysis.

According to Table 2, the water content data collected in Poblado's mudflows vary between 28.8 and 62%, which can be attributed to the fact that this is one of the properties with the greatest dispersion according to the Cv. The causes of this variation are threefold: (1) the different positions of the water table; (2) inclusion of rock fragments in the initial moisture weight; (3) the combination with organic material reported during the site investigation.

The low dispersion of the overall data, except for water content, may be indicative of the mudflow matrix's homogeneity, which might be explained by the landslide formation process of this slope's deposits. The origin landslides can produce a mixture of mineral and grain sizes at the same weathering degree, forming a homogeneous soil matrix with some less weathered rock fragments and organic soils.

Residual soils results

A total of 29 samples were analysed from San Diego IC residual soils. We found they had a predominant classification as ML (silt), as shown in Figure 6. Nonetheless, there is a clear difference in the histograms when compared with mudflows. The differences between mudflows and residual soils may be related to the changes in the state of weathering in residual soils. The variation of the state of weathering with depth also varies with the grain size distribution throughout soil profile.

Determining the weathering profile is one of the most complex tasks in studying residual soils. This part is usually carried out using the state of weathering categorization (e.g. Deere and Patton 1971). These categories have sharp changes and transitions between them are not considered, so it is common to expect a nonuniform trend in the analysis of the grain size classification according to the USCS.

Based on the results shown in Table 3, the Cv of the index properties for the residual soils has values from medium to high (Cv between 21 and 35%) for the same state of weathering. These ranges possibly indicate the susceptibility of these properties to grain size distribution, which in turn depends on the mineralogical concentration and the possible mixture of soils with different degrees of weathering.

Standard penetration test (SPT) data

To prevent the measurement and human performance uncertainties in the geological statistical analyses, the same SPT equipment was used for both geologies following the ASTM D1586 (2011) standards. The obtained N values were corrected to obtain the $(N_1)_{60}$. Because the analysed soils show a fine-grained grain size distribution, the overburden stress correction (C_N) was not carried out according to Skempton (1986).

The $(N_1)_{60}$ can have different limitations owing to the susceptibility of this value to random factors such as rock fragments

Table 2. *Index properties results of Poblado mudflows (adapted from Viviescas and Osorio 2015)*

	LL (%)	PL (%)	w (%)	USCS	γ_m (kN/m^3)	Gs	Saturation (%)
Average	61.9	41.7	49.2	MH	16.5	2.72	90.18
Maximum value	80.0	53.0	62.0		17.6	2.75	90.0
Minimum value	44.0	25.0	28.8		15.7	2.56	80.0
Standard deviation	11.39	7.31	11.88		0.4	0.13	5.33
Cv (%)	18	18	24		2	5	6

LL, liquid limit; PL, plastic limit; w, water content; USCS, Unified Soil Classification System; Gs, specific gravity of solids; γ_m, moist unit weight.

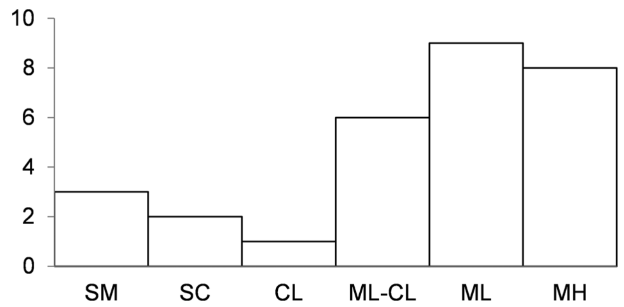


Fig. 6. Frequency histogram of the USCS for San Diego residual soil.

content, variations in the state of weathering and human errors. Therefore, using a similar method to that of Viviescas *et al.* (2019), we performed a previous cluster analysis to identify the main trend outliers for their removal. Cluster analysis is a generic term for a wide range of numerical methods for examining multivariate data to uncover or discover group observations with similar characteristics (Everitt and Hothorn 2011). Therefore, the previously implemented cluster analyses ease the influence of the presence of SPT outliers in the data analyses, as a way to formalize what human observers do with the removal of outliers through an objective mathematical method. The results of the variation with depth of the $(N_1)_{60}$ according to the analysed geology are presented in Figure 7.

$(N_1)_{60}$ statistical characterization

We performed a Shapiro–Wilk test and a Kolmogorov–Smirnov test (explained at the Appendix) for the definition of the best $(N_1)_{60}$ PDF for each geology. Table 4 and Figure 8 show that mudflows present a gamma PDF (with lognormal tendency) and the IC residual soils present a logistic function (with normal tendency). Table 4 and Figure 8 show the possible influence of the geological origin in the PDF behaviour of the field tests, even with similar $(N_1)_{60}$ mean values.

SPT test correlations

The SPT is used to indirectly determine the soil undrained compressive strength (q_u) and the peak effective friction angle (ϕ'), which can be difficult to obtain owing to the absence of laboratory tests and the difficulty in obtaining undisturbed samples. The correlations for different properties are widely found in the literature, but some of them lack information on the correlation coefficient (R^2) and the results of the regression analyses (Sivrikaya and Toğrol 2006). The peak effective friction angle and the undrained compressive strength are obtained through the replacement of various correlations, as shown in Tables 5 and 6.

According to Clayton (1995), the SPT–N correlations cannot be considered as an accurate measure of the shear strength properties owing to the empirical procedures in the formulation and the lack of clarity of the SPT–N value's standardization (no definition for the energy of future corrections). However, McGregor and Duncan (1998) suggested that the $(N_1)_{60}$ was apparently implemented in the correlations instead of the field N value because the hammers most commonly used at the time transmitted a theoretical energy of 60% for the test; however, no statistical information has been reported.

The absence of the initial statistical analysis of the correlations means that we cannot evaluate the uncertainties of the mathematical formulations and the limitations for their use. Although Bayesian methods have been developed to mitigate the absence of the statistical details of the correlations (e.g. Wang and Zhao 2016), a statistical evaluation of the correlations according to the geological origin was made to reduce the uncertainty in the shear strength correlations.

Estimated peak effective friction angle results

We evaluated the Cv and the PDF of the correlated ϕ' according to the $(N_1)_{60}$ PDF on mudflows and residual soils. We performed this evaluation by replacing each corrected N value according to the correlations shown in Table 5. We then performed a χ^2 and Shapiro–Wilk goodness-of-fit test to evaluate the normality hypothesis, as shown in Tables 7 and 8. We compared these results with the PDF in the literature to determine the influence of the SPT–N PDF in the shear strength functions.

According to Table 7, we found that seven out of 12 correlations for the mudflows tend to present a normal PDF (e.g. the correlations of Hatanaka and Uchida (1996), Japan Road Bureau (1984), Kishida (1969), Shioi and Fukui (1982), and Kulhawy and Mayne (1990). The previous correlations correspond to those functions that present a square root of the $(N_1)_{60}$. According to Table 8, six correlations for the residual soil, on the other hand, have a normal tendency (Japan National Railway (1984), Japanese Railway Standards (1984),

Table 3. *Index properties results of San Diego IC residual soils*

	LL (%)	PL (%)	w (%)	USCS	γ_m (kN/m^3)	Gs	Saturation (%)
Average	46.1	30.5	27.6	ML	17.9	2.67	81.80
Maximum value	73.5	45.2	51.1		18.3	2.70	84.0
Minimum value	28.6	17.3	8.6		16.8	2.65	67.0
Standard deviation	11.60	6.38	9.71		0.7	0.02	11.12
Cv (%)	25	21	35		4	1	14

LL, liquid limit; PL, plastic limit; w, water content; USCS, Unified Soil Classification System; Gs, specific gravity of solids; γ_m, moist unit weight.

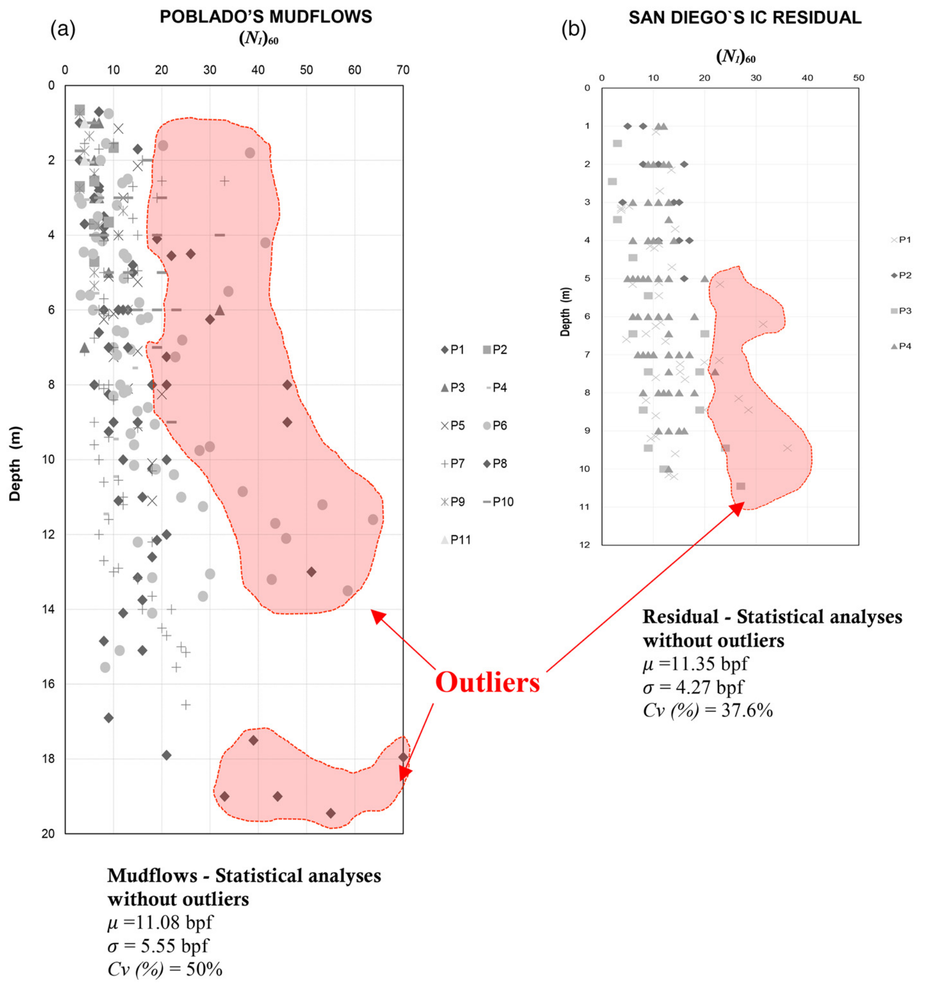


Fig. 7. Scatter plot of the $(N_1)_{60}$ variation with depth. (**a**) Poblado mudflows. (**b**) IC residual soils from the San Diego Stock. P refers to projects.

Table 4. *Shapiro–Wilk normality test and a Kolgomorv–Simirnov test for the $(N_1)_{60}$ PDF goodness-of-fit*

Geology	Shapiro–Wilk			Kolmogorov–Smirnov (*P* value)				
	Statistic	*P* value	Analysis	Normal	Lognormal	Weibull	Gamma	Logistic
Mudflows	0.932	0.00	Reject normality	—	0.274	0.207	0.493	—
Residual	0.97	0.064	Normality cannot be rejected	0.248	0.024	0.297	—	0.506

Kulhawy and Mayne (1990), Peck *et al.* (1974) and Wolff (1989)). Unlike the mudflows, the residual soil correlations, which are closer to a normal tendency, are those with polynomial, tangential or linear formulations. Therefore, it is possible to infer that to achieve a normal or lognormal PDF of the peak effective friction angle, the selection of the ϕ'–$(N_1)_{60}$ correlations depends on the geological origin.

In general, the Kulhawy and Mayne (1990) correlation is the only one that shows a normal behaviour for both geologies owing to the influence of the effective stress on the correlation, as this is the only expression that takes that property into account.

Estimated undrained compressive strength

Following what has been done in the effective parameters section, we performed an evaluation of the Cv and the PDF of the undrained compressive strength (q_u) by replacing each corrected N value

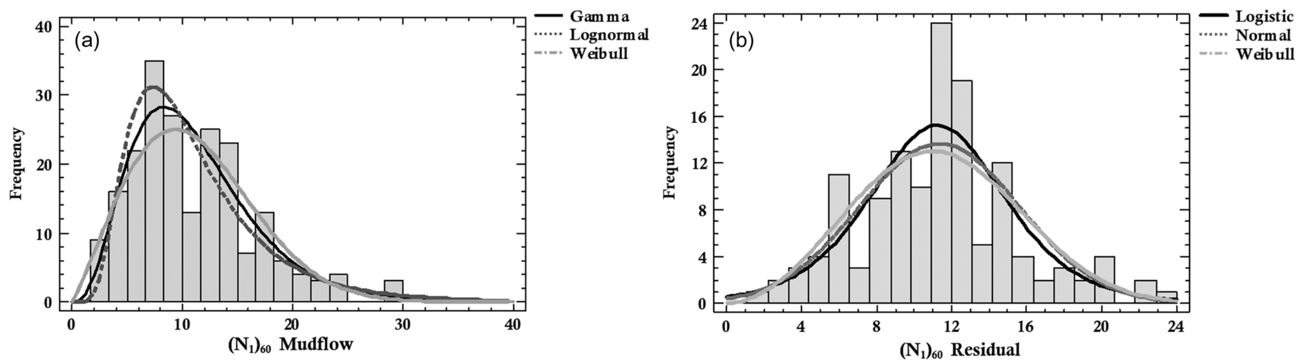


Fig. 8. Histogram of the $(N_1)_{60}$ variation with depth for the (**a**) Poblado mudflows and (**b**) IC residual soils from the San Diego Stock.

Table 5. *Friction angle ϕ' correlations from SPT tests*

Author	Correlations*	Equation	Reference
Kulhawy and Mayne (1990)	$\phi' = \tan^{-1}\left[\dfrac{N_{60}}{12.2+20.3\left(\dfrac{\sigma'}{Pa}\right)}\right]^{0.34}$	2	Kulhawy and Mayne (1990)
Japanese Railway Standards (JRS) (1984)	$\phi' = 0.42N_{60} + 27°$	3	Bowles (1996)
Shioi and Fukui (1982)	$\phi' = \sqrt{15(N_1)_{60}} + 15°$	4	Bowles (1996)
Wolff (1989)	$\phi' = 27.1° + 0.3(N_1)_{60} - 0.00054[(N_1)_{60}]^2$	5	Wolff (1989)
Hatanaka and Uchida (1996)	$\phi' = \sqrt{20(N_1)_{60}} + 20$	6	Hatanaka and Uchida (1996)
Peck *et al.* (1974)	$\phi' = 28.5° + 0.188(N_1)_{60}$	7	Gonzales (1999)
Kishida (1969)	$\phi' = 15° + \sqrt{24(N_1)_{60}}$	8	Gonzales (1999)
Muromachi (1974)	$\phi' = 20° + 3.5°[(N_1)_{60}]^{1/2}$	9	Décourt (1990)
Schmertmann (1975)	$\phi' = \tan^{-1}\left\{\left[\dfrac{(N_1)_{60}}{32.5}\right]^{0.34}\right\}$	10	Schmertmann (1975)
Peck *et al.* (1974)	$\phi' = 26.25° \times \left\{2 - \exp\left[-\dfrac{3(N_1)_{60}}{248}\right]\right\}$	11	Gonzales (1999)
Japan National Railway (JNR) (1984)	$\phi' = 27° + 0.36(N_1)_{60}$	12	Shioi and Fukui (1982)
Japan Road Bureau (JRB) (1984)	$\phi' = 15° + [18(N_1)_{60}]^{0.5}$	13	Hatanaka and Uchida (1996)

σ', effective vertical stress; *Pa*, atmospheric pressure.*Some of the correlations, such as that of Hatanaka and Uchida (1996), were obtained by relating the *N* value to the direct measurements of ϕ' in triaxial tests performed on frozen natural sand samples (Brown *et al.* 2010).

Table 6. *Correlations between SPT–N and q_u according to soil types in fine-grained soils (adapted from Sivrikaya and Toğrol 2006)*

Author	Soil type	q_u (kPa)	Equation
Sanglerat (1972)	Clay	$25N$	14
	Silty clay	$20N$	15
Terzaghi and Peck (1967)	Fine-grained soil	$12.5N$	16
Sowers (1979)	High-plasticity clay	$25N$	17
	Medium-plasticity clay	$15N$	18
	Low-plasticity clay and silt	$7.5N$	19
Nixon (1982)	Clay	$24N$	20
Hara *et al.* (1974)	Fine-grained soil	$58N^{0.72}$ $R = 0.865$	21
	$n = 180$		
Sivrikaya and Toğrol (2002)	CH	$9.7N_{\text{field}}$ $R = 0.83$	22
	$n = 113$	$13.63N_{60}$ $R = 0.80$	
	CL	$6.7N_{\text{field}}$ $R = 0.76$	23
	$n = 72$	$9.85N_{60}$ $R = 0.73$	
	Fine-grained soil	$8.64N_{\text{field}}$ $R = 0.8$	24
	$n = 226$	$12.36N_{60}$ $R = 0.78$	
	Fine-grained soil	$(0.19I_p + 6.2)N_{60}$,	25
	$n = 30$	N_{60} <25	
Sivrikaya and Toğrol (2006)*	CH	$11N_{\text{field}}$ $R = 0.80–0.86$	26
	$n = 206$	$15.6N_{60}$ $R = 0.80–0.87$	
	CL	$7.4N_{\text{field}}$ $R = 0.75–0.82$	27
	$n = 150$	$10.7N_{60}$ $R = 0.73–0.83$	
	Clay	$9.5N_{\text{field}}$ $R = 0.73–0.82$	28
	$n = 356$	$13.8N_{60}$ $R = 0.75–0.77$	
	Fine-grained soil	$8.9N_{\text{field}}$ $R = 0.72–0.80$	29
	$n = 478$	$12.7N_{60}$ $R = 0.71–0.78$	

R, coefficient of determination; *n*, number of data.*The *R* values range refers to the overall linear regressions obtained by different data and procedures from the same soil reported by Sivrikaya and Toğrol (2006).

according to the q_u correlations in Table 6. The correlations selected were those that were closest to the grain size distribution of each of the geologies. The mudflows were classified according to the USCS as MH, so they are similar to the CL and fine-grained correlation as shown in Table 6. Residual soils are classified as ML, so they were analysed as a fine-grained soil and with medium- to low-plasticity clay correlations as shown in Tables 9 and 10. We performed a χ^2 test and Shapiro–Wilk goodness-of-fit test to evaluate the hypothesis of normality of the correlated q_u.

According to the results in Tables 9 and 10, the PDF of the undrained compressive strength preserves the $(N_1)_{60}$ input variable function. For the mudflows, we found that the q_u presents functions that tend to be a not-normal PDF, and the residual soils present a logistic–normal function according to the $(N_1)_{60}$ PDF as shown in Figure 8.

Shear strength laboratory results

The shear strength laboratory tests conducted on the undisturbed soil samples recovered at the two distinct geological formations included a consolidated drained (CD) and consolidated undrained (CU) direct shear test (ASTM D3080 2014), unconfined

Table 7. *Goodness-of-fit test to evaluate the hypothesis of normality for the correlated effective friction angle on the mudflows*

			Normal goodness-of fit test				
			χ^2		Shapiro–Wilk		
Correlation	Mean ϕ'	Standard deviation	Statistic	*P* value	Statistic	*P* value	Hypothesis
Hatanaka and Uchida (1996)	34.43	3.68	11.21	0.082	0.966	0.002	Normal–not normal
Japan National Railway (JNR) (1984)	30.99	1.996	19.87	0.003	0.932	0.000	Not normal
Japan Road Bureau (JRB) (1984)	28.68	3.49	12.24	0.057	0.966	0.002	Normal–not normal
Japanese Railway Standards (JRS) (1984)	31.65	2.33	19.88	0.003	0.932	0.000	Not normal
Kishida (1969)	30.80	4.03	11.21	0.082	0.966	0.003	Normal–not normal
Kulhawy and Mayne (1990)	33.77	3.71	4.92	0.555	0.977	0.136	Normal
Muromachi (1974)	31.30	2.88	10.84	0.094	0.966	0.003	Normal–not normal
Peck *et al.* (1974)	30.58	1.04	19.88	0.003	0.932	0.000	Not normal
Peck *et al.* (1974)	29.49	1.50	26.54	0.0002	0.943	0.000	Not normal
Schmertmann (1975)	33.36	4.42	7.63	0.266	0.962	0.001	Normal–not normal
Shioi and Fukui (1982)	29.78	3.77	11.87	0.065	0.966	0.003	Normal–not normal
Wolff (1989)	30.34	1.58	20.92	0.002	0.936	0.000	Not normal

Table 8. *Goodness-of-fit test to evaluate the hypothesis of normality for the correlated effective friction angle on the residual soils*

			Normal goodness-of-fit test				
			χ^2		Shapiro–Wilk		
Correlation	Mean ϕ'	Standard deviation	Statistic	*P* value	Statistic	*P* value	Hypothesis
Hatanaka and Uchida (1996)	34.31	2.57	21.04	0.001	0.974	0.175	Not normal–normal
Japan National Railway (JNR) (1984)	30.80	1.25	10.33	0.066	0.969	0.064	Normal
Japan Road Bureau (JRB) (1984)	28.57	2.44	21.04	0.001	0.974	0.174	Not normal–normal
Japanese Railway Standards (1984)	31.44	1.45	10.33	0.066	0.969	0.064	Normal
Kishida (1969)	30.67	2.81	17.12	0.004	0.974	0.175	Not normal–normal
Kulhawy and Mayne (1990)	32.89	4.33	2.72	0.743	0.969	0.061	Normal
Muromachi (1974)	31.20	2.01	21.04	0.001	0.974	0.176	Not normal–normal
Peck *et al.* (1974)	30.48	0.65	10.33	0.066	0.969	0.062	Normal
Peck *et al.* (1974)	29.38	0.97	10.64	0.059	0.973	0.149	Normal
Schmertmann (1975)	33.458	3.34	38.01	0.000	0.946	0.000	Not normal
Shioi and Fukui (1982)	29.661	2.63	21.84	0.001	0.974	0.174	Not normal–normal
Wolff (1989)	30.2	1.00	7.56	0.182	0.971	0.087	Normal

compression test (ASTM-D2166/D2166M-16 2016) and triaxial test (ASTM-D7181 2011). Table 11 presents a summary of the shear strength tests for each geological unit.

Laboratory effective shear strength analysis according to correlations

Because of the lack of laboratory peak effective friction angle (ϕ') results for each measured *N* value, we performed a comparative analysis based on the Mohr–Coulomb soil shear strength criteria (normal stress v. shear stress). The analysis compares the laboratory CD shear strength results and the shear strength obtained using the correlated effective friction angle (ϕ'_c) to evaluate which ϕ'–$(N_1)_{60}$ correlations most closely resemble the drained laboratory tests results. The procedure consists in replacing each corrected *N* value in the respective ϕ' correlation with the obtained ϕ'_c. Finally, the ϕ'_c was replaced in equation (2) along with the respective normal stress to obtain the effective shear strength (S'):

$$S' = \sigma'_n \tan \phi'_c \tag{2}$$

Table 9. *Mean values, standard deviation and goodness-of-fit test to evaluate the hypothesis of normality for the correlated undrained compressive strength for the mudflows*

			Normal goodness-of-fit test				
			χ^2		Shapiro–Wilk		
Correlation	Mean q_u (kPa)	Standard deviation	Statistic	*P* value	Statistic	*P* value	Hypothesis
Sanglerat (1972) (silty clay)	221.55	110.91					Not normal
Terzaghi and Peck (1967) (fine-grained soils)	138.47	69.32					Not normal
Sowers (1979) (low-plasticity clay and silt)	83.08	41.59					Not normal
Sivrikaya and Toğrol (2002) (low-plasticity clay)	109.11	54.62	19.876	0.0029	0.9316	9.98757×10^{-13}	Not normal
Sivrikaya and Toğrol (2002) (fine-grained soils)	137.16	68.54					Not normal
Sivrikaya and Toğrol (2006) (low-plasticity clay)	118.53	59.34					Not normal
Sivrikaya and Toğrol (2006) (fine-grained soils)	140.68	70.428					Not normal

Table 10. *Mean values, standard deviation and goodness-of-fit test to evaluate the hypothesis of normality for the correlated undrained compressive strength for the residual soils*

Correlation	Mean q_u (kPa)	Standard deviation	Normal goodness-of-fit test				Hypothesis
			χ^2		Shapiro–Wilk		
			Statistic	P value	Statistic	P value	
Terzaghi and Peck (1967) (fine-grained soils)	141.87	54.15	7.828	0.251	0.9695	0.0644	Normal
Sowers (1979) (medium plasticity clay)	85.12	31.99					Normal
Sivrikaya and Toğrol (2002) (fine-grained soils)	140.28	52.73					Normal
Sivrikaya and Toğrol (2002) (CL)	111.79	42.022					Normal
Sivrikaya and Toğrol (2006) (fine-grained soils)	144.14	54.18					Normal
Sivrikaya and Toğrol (2006) (CL)	121.4	45.65					Normal

Table 11. *Number of shear strength tests evaluated on undisturbed samples for each geological unit according to the type of stress*

Location, geology	Type of stress	Number of data		Type of test
		Reference	Number	
Poblado mudflows	Drained	Parra and Hidalgo (2015)	78	CD direct shear test
		Current study	3	CD direct shear
			1	CU triaxial
	Undrained	Current study	5	Unconfined compression
			4	CU direct shear test
			3	UU direct shear test
			1	CU triaxial
San Diego IC residual	Drained	Current study	3	CD direct shear strength
	Undrained	Current study	2	UU triaxial test
			7	UU direct shear

where σ'_n is effective normal stress at the location of each measured N value and ϕ'_c is correlated effective friction angle.

The comparative shear strength analyses obtained from correlations and the drained laboratory results shown in Table 11 for both geologies are presented in Figures 9 and 10. The overall laboratory results show a consistent linear stress increment between 10 and 370 kPa normal stress. The comparative results show that the shear strength predicted by N overestimates, to some extent, the laboratory shear strength. This is due to imprecision of the ϕ' correlations for silt soils as indicated by the significant differences of the results obtained, even with 'closer' correlations such as that of Kulhawy and Mayne (1990).

The overall analysis of the effective shear stress linear regressions obtained from the correlations shows the presence of negative effective cohesion caused by the presence of high ϕ'_c based on the laboratory results. According to the regression analysis of the correlations, the effective shear strength obtained by all the correlations has a high correlation coefficient (R^2). However, the regression results for the correlations do not show similarity to the laboratory results.

The residual soil results show that most of the effective shear stress of the correlations is within the confidence intervals of the laboratory results. Residual soils have negative cohesion confidence intervals in the lower but not the upper range, with a tendency to reach 0.0 (P value >0.05, $C' \approx 0$). This may indicate the influence of the grain size distribution on the effective shear strength analysis. Soils with low plasticity (residual soils) have more values within the confidence intervals, whereas the results for those with higher plasticity (mudflows) depart from this range considerably. This outcome could be explained as the low-plasticity soils tend to behave as sand, so they are more related to the SPT–N correlations; furthermore, high-plasticity soils tend to exhibit clay shear strength

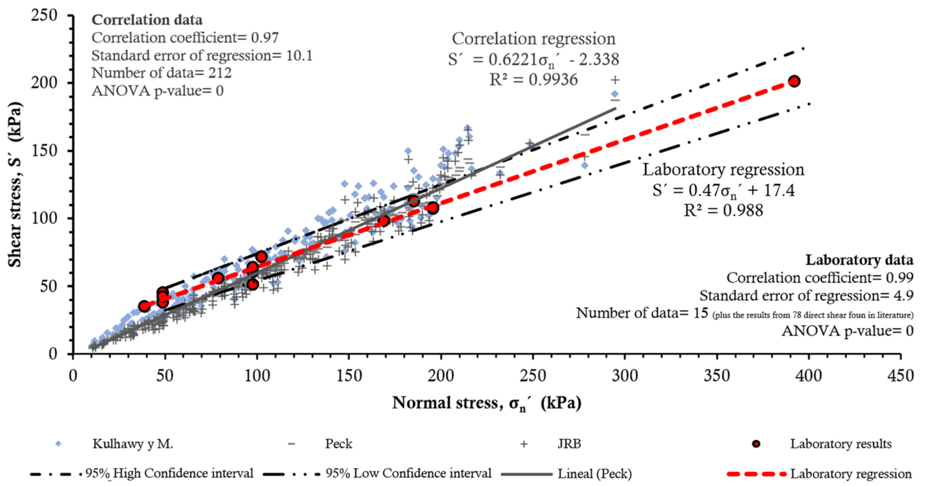


Fig. 9. Comparative analyses from overall correlated effective shear stress and laboratory CD direct shear tests for mudflows.

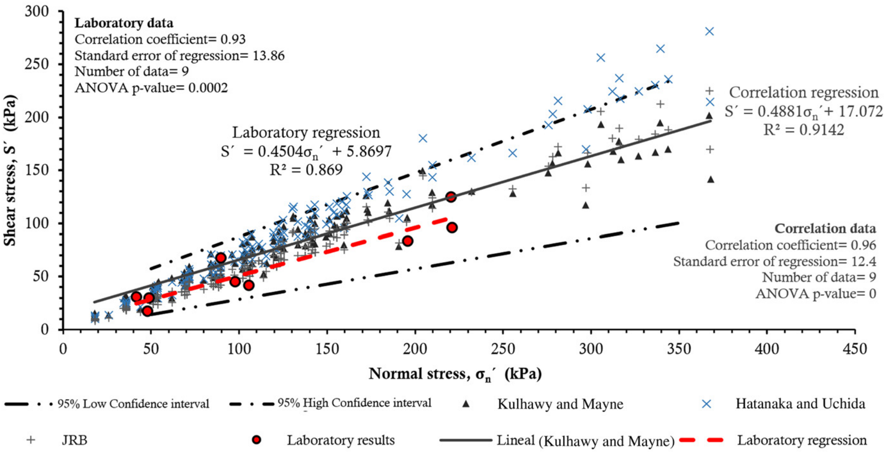


Fig. 10. Comparative analyses from overall correlated effective shear stress and laboratory CD direct shear tests for residual soils.

behaviour (Penman 1953; Brandon *et al.* 2006). Therefore, high-plasticity soils can show essential inconsistencies in SPT–*N* correlations, which were mainly made for sand and gravel.

The above analysis suggests the non-use of the SPT–*N* correlations for high-plasticity and fine-grained soils. Therefore, the magnitude depends on the predominant grain size distribution regardless of the geological context. The results also show that defining a unique friction angle correlation for each geological soil is ambiguous and can lead to essential uncertainties in geotechnical designs (e.g. Viviescas *et al.* 2020).

Laboratory undrained compressive strength analysis according to correlations

We performed a regression analysis of the undrained laboratory results to define the confidence intervals for the undrained parameters. We then performed a comparative graphical analysis to evaluate which correlations of the undrained compressive strength most closely resemble the properties obtained through the undrained laboratory tests.

The comparative analyses showed that most of the correlations are far from the laboratory results, with the exception of the Sowers (1979) correlation, as shown in Figures 11 and 12. The laboratory and correlation inconsistencies are due to the fact that the correlations were mostly made for clay. Therefore, the Sowers correlation is the only one that really coincides with the grain size distribution of the analysed soils.

The Sowers analysis for the mudflows and residual soils reveals that although the correlation is similar to the laboratory results for the mudflows, the correlated q_u values still show a significant number of data outside the confidence intervals. However, most of the correlated q_u values in the residual soils are within the 95% confidence range of laboratory results.

Definition of the q_u*–N correlation according to the results for each geology*

Prior to defining the q_u PDF for the analysed geologies, we performed an evaluation of the $q_u = a(N_1)_{60}$ correlation according to the obtained laboratory and *N* results. We determined the correlation through a regression analysis of the measured q_u obtained through laboratory tests versus the corresponding $(N_1)_{60}$ as shown in Figure 13. The results show a similar correlation to that of Sowers (1979), where residual soils have a higher constant than mudflows, even when the residual soils have a grain size distribution similar to that of sands.

PDF adjustment of q_u

Once we identified the q_u correlation for each geology, we compared the laboratory, the closest correlation (Sowers 1979) and the new SPT–*N* correlation PDFs for each geology. However, owing to the lack of laboratory tests to represent a meaningful statistical q_u–*N* correlation, we evaluate the accuracy of the correlations in obtaining

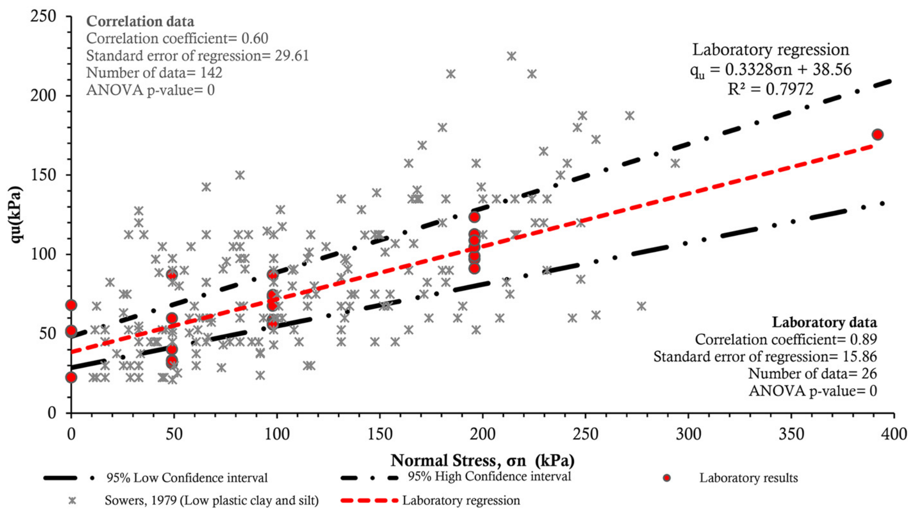


Fig. 11. Comparative analyses between Sowers (1979) correlated undrained compressive strength and laboratory results in mudflows.

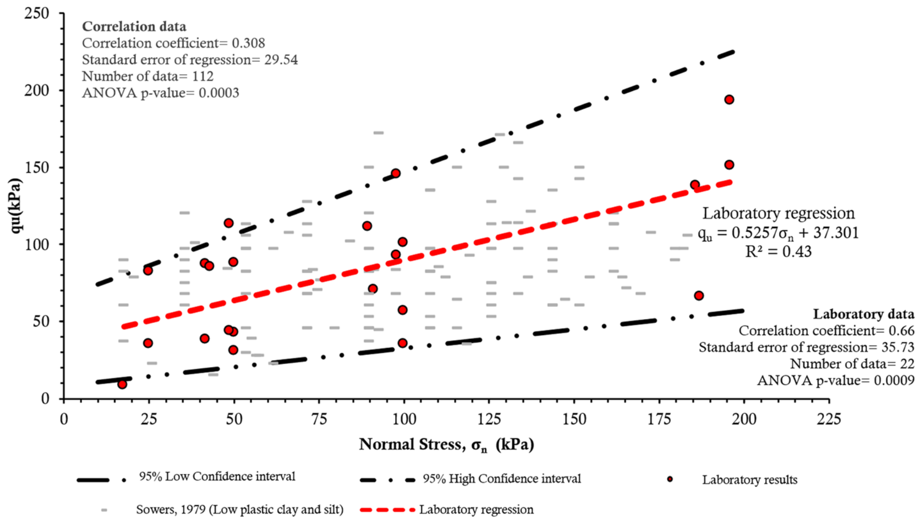


Fig. 12. Comparative analyses between Sowers (1979) correlated undrained compressive strength and laboratory results in residual soils.

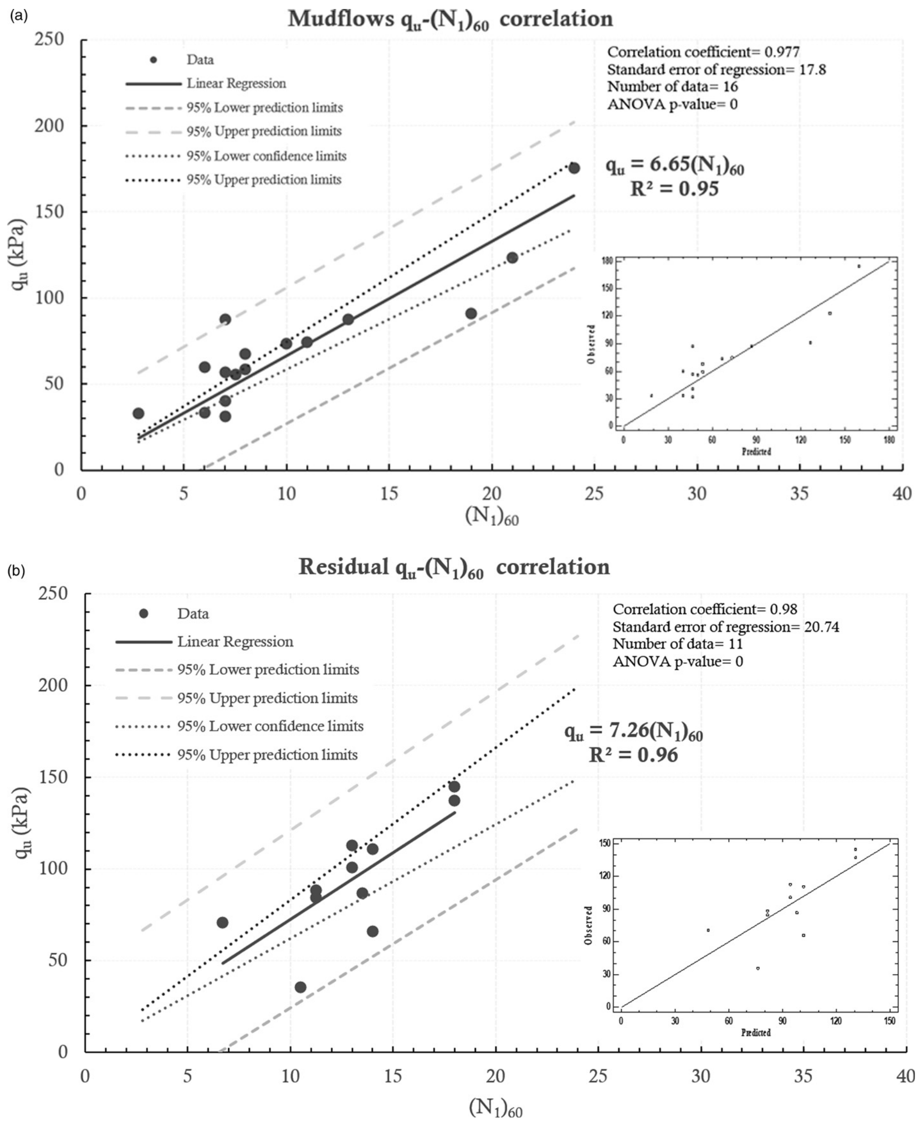


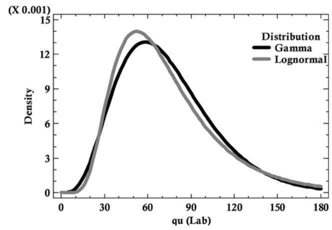
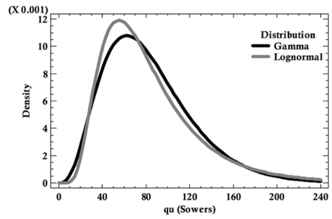
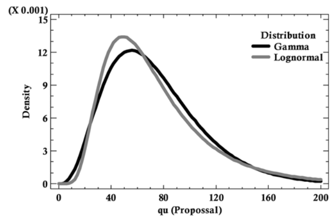
Fig. 13. Regression analyses of the measured undrained compressive strength (q_u) and their corresponding $(N_1)_{60}$ values for the definition of the $q_u = aN$ correlation on mudflows (MH) and residual soils (ML).

similar PDFs to the laboratory results on each geology to assess the statistical meaningfulness of the obtained correlation.

Mudflow soils q_u *PDF analysis*

According to the Kolmogorov–Smirnov test in Table 12, it is evident that the laboratory and the q_u correlations have a similar PDF (gamma–lognormal). Taking this into consideration, the PDF analysis shows that mudflows present a similar Cv between the laboratory and the correlations. However, the ranges and means show that the Sowers correlation must be adjusted to give similar values to the laboratory results. Therefore, the evaluation with the adjusted correlation obtained from the regression analyses ($q_u = 6.65(N_1)_{60}$) shows values close to the laboratory's minimum, maximum, average and Cv. Hence, this new correlation is also capable of defining the q_u PDF.

Table 12. *Analyses of the adjusted PDF of the correlated undrained compressive strength and laboratory results for the mudflows soils*

Method	Minimum (kPa)	Maximum (kPa)	Average (kPa)	Cv (%)	PDF evaluation (Kolmogorov–Smirnov)	*P* value	PDF parameters*	Probability density function (PDF)
Laboratory results	22.5	175.42	73.71	47	**Gamma**	**0.9896**	Gamma: shape = 4.821, scale = 0.0654	
					Logistic	0.9245		
					Lognormal	**0.9468**	Lognormal: μ = 74.45, σ = 38.28	
					Normal	0.8609		
					Weibull	0.9824		
Correlation Sowers (1979) (low-plasticity clay and silt) $q_u = 7.5\ (N_1)_{60}$	21.09	225.0	83.08	50	**Gamma**	**0.4931**	Gamma: shape = 4.001, scale = 0.0482	
					Logistic	0.0737		
					Lognormal	**0.2738**	Lognormal: μ = 83.88, σ = 47.61	
					Normal	0.0150		
					Weibull	0.2069		
Current study correlation $q_u = 6.65\ (N_1)_{60}$	18.7	199.5	73.7	50	**Gamma**	**0.4931**	Gamma: shape = 4.001, scale = 0.0543	
					Logistic	0.0737		
					Lognormal	**0.2738**	Lognormal: μ = 74.37, σ = 42.22	
					Normal	0.0150		
					Weibull	0.2069		

Cv, coefficient of variation.*The PDF parameters are the values that control the shape and distribution of the probability function. Values in bold represent the graphed probability functions.

Residual soil q_u PDF analysis

The Shapiro–Wilk analyses for the laboratory and correlation results have a PDF with a normal tendency (*P* value = 0.4618 for laboratory tests; *P* value = 0.0644 for the Sowers correlation). However, Table 13 shows that the correlations and laboratory tests have a Weibull and logistic PDF. The differences between the two methods are related to the Cv variation between the laboratory tests and the correlations.

We performed a PDF analysis on the residual soils, the results of which are given in Table 13. We found that the Sowers correlation is very close to the average values but with more biased minimum and maximum ranges owing to the low $(N_1)_{60}$ Cv for the laboratory results. However, the adjusted correlation obtained from the regression analyses ($q_u = 7.26(N_1)_{60}$) shows a better PDF fit, as was shown with the mudflow soils. Therefore, the obtained q_u–N correlation for residual soils is also capable of giving an accurate q_u PDF.

Other previously published results

The database used in this paper has been used by the authors for different analyses published in other papers. The main results obtained in these papers are described below.

Shear strength variation with depth

Viviescas *et al.* (2019) found that that the undrained shear strength showed a z^2 (where z is depth in metres) variation with depth instead of a linear tendency. The z^2 function for residual soils is about twice that of the mudflows, owing to the decrease in the state of weathering with depth. These results show that mudflows are highly random soils, which makes an in-depth characterization difficult. This is because the mudflow shear strength tendency depends on the conditions that generated the landslide of origin and the geomorphology of each of the projects at the time of soil formation. Therefore, residual soils show a better in-depth shear strength characterization because they are influenced only by the processes of weathering and not by erosion or by transportation.

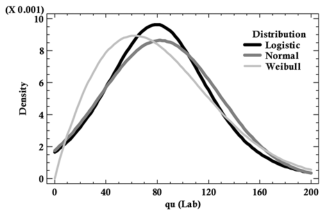
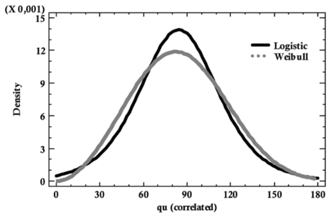
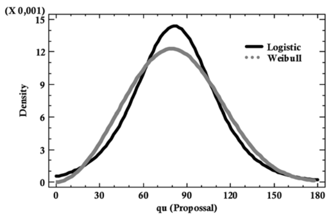
Spatial variability

Viviescas *et al.* (2021) found that the soil's geological origin can be an essential aspect in the spatial variability of soils measured by the spatial correlation length (θ). The spatial correlation length is an important property in reliability-based design, in addition to the mean and standard deviation. The published results show that the average horizontal correlation length (θ_H) for mudflows is $\theta_H \approx 6.0$ m and for residual soils $\theta_H \approx 20$ m. Based on these results, the θ_H for residual soils is approximately three times the mudflows' horizontal length. Therefore, it is inferred from the results in the published paper that the geological influence on the θ_H magnitude is related to the geological processes that formed the soils. Materials with abrupt changes (mudflows) will have lower θ_H compared with stationary soils (residual soils).

Summary and discussion

- The results for index properties indicate that mudflows are less variable than residual soils, which may be explained by

Table 13. *Analyses of the adjusted PDF of the correlated undrained compressive strength and laboratory results for IC residual soils*

Method	Minimum (kPa)	Maximum (kPa)	Average (kPa)	Cv (%)	PDF evaluation (Kolmogorov–Smirnov)	*P* value	PDF	PDF parameters*	Probability density function (PDF)
Laboratory results	8.63	193.06	82.32	56	Gamma	0.8967	Weibull–logistic normal	Weibull: shape = 1.894, scale = 92.74	
					Logistic	**0.9090**			
					Lognormal	0.7081			
					Normal	**0.9085**			
					Weibull	**0.9558**			
Correlation Sowers (1979) (low-plasticity clay and silt) $q_u = 7.5\ (N_1)_{60}$	15	172.13	85.12	38	Gamma	0.1074	Logistic–Weibull		
					Logistic	**0.5062**		Logistic: α = 85.12, β = 17.83	
					Lognormal	0.0237			
					Normal	0.2483			
					Weibull	**0.2973**		Weibull: shape = 2.876, scale = 95.46	
Current study correlation $q_u = 7.25\ (N_1)_{60}$	14.5	166.39	82.3	38	Gamma	0.1074	Logistic–Weibull		
					Logistic	**0.5062**		Logistic: α = 82.3, β = 17.24	
					Lognormal	0.0237			
					Normal	0.2483			
					Weibull	**0.2973**		Weibull: shape = 2.876, scale = 92.281	

Cv, coefficient of variation.*The PDF parameters are the values that control the shape and distribution of the probability function.

their respective geological origin. Mudflows were formed by landslides, which generated a mixture of soil grain sizes, and have the same weathering degree as some less weathered rock fragments. On the other hand, residual soils have a greater mineralogical heterogeneity owing to the variability in the state of weathering with depth and corresponding grain size distribution.

- The homogeneity of index properties should not be considered an indicator for shear strength isotropy, as it is explained by the important uncertainties in the mudflow shear strength. Mudflow shear strength uncertainty could be related to the variation in void ratio according to the location of the deposit. Void ratio variation in mudflows depends on the energy of the process of the soil's formation, the initial erosion processes and previous characteristics of the deposit site.
- According to the comparisons between the shear resistance predicted by N and values obtained from the laboratory tests, for low- and high-plasticity silt it may be inferred that the peak friction angle depends on the predominant grain size distribution of the soil regardless of the soil's origin. Therefore, the ϕ'–$(N_1)_{60}$ correlations should be used only for sand and gravel. However, to achieve a normal or lognormal PDF of the peak effective friction angle, the selection of the ϕ'–$(N_1)_{60}$ correlations could be dependent on the geological origin.
- We obtained a new q_u–$(N_1)_{60}$ correlation that corresponds to the geological origin (mudflows and residual soils). It is shown that the input PDF of the $(N_1)_{60}$ coincides with the output of the q_u correlations. Therefore, the capability of the new correlations to obtain an accurate PDF according to the laboratory results is shown. We conclude that the new correlations are capable of obtaining an accurate q_u value and PDF.
- The q_u analyses show that geology may influence the shape characteristics of the PDF. Mudflows have a q_u PDF with a lognormal tendency possibly caused by the presence of rock fragments and randomness related to the soil formation. However, residual soils under the same state of weathering tend to have a normal q_u PDF, probably owing to the stationary origin of these soils (which have never been transported).
- The comparison between the $(N_1)_{60}$ and the laboratory PDFs shows that the undrained soil behaviour has a distribution similar to the SPT. This is because the SPT is a rapidly performed test that does not allow drainage at any time in accordance with the undrained parameters.

Conclusions

To evaluate the influence of the geological origin on the variability of geotechnical properties, we analysed geology with abrupt changes and within a stationary environment. Mudflows are soils formed by previous landslides, which generate a mixture of grain sizes with the same weathering degree as some less weathered rock fragments. Therefore, mudflows show lower index property variability owing to the mixture of the soil matrix. In terms of shear strength, a gamma–lognormal q_u PDF is expected, possibly owing to the presence of rock fragments and the void ratio variation according to the location of the deposit.

On the other hand, the residual soils (stationary soils), which were formed from the weathering of the *in situ* rock with the presence of mineral concentration, show higher index property variability in comparison with the mudflows. However, a logistic–Weibull q_u PDF is expected in residual soils because of the low shear strength variation that occurs throughout the same state of weathering.

Finally, we conclude that the shape of the PDF (effective and total parameters) depends on the soil formation characteristics described by the geology. However, the magnitude and variation of ϕ' depend on the dispersion and predominant soil grain size distribution.

Acknowledgements The authors would like to acknowledge the financial support to this research project under the National Doctoral Grant Scheme No. 727 of 2015, provided by the Administrative Department of Science, Technology, and Innovation of Colombia - Colciencias.

Author contributions **JCV**: conceptualization (lead), investigation (lead), methodology (equal), validation (lead), writing – original draft (lead); **JPO**: conceptualization (supporting), methodology (equal), supervision (equal), writing – review & editing (equal); **CP**: conceptualization (supporting), methodology (supporting), supervision (supporting), writing – review & editing (supporting)

Funding This work was funded by the Administrative Department of Science, Technology and Innovation of Colombia – Colciencias (727-2015).

Data availability All data generated or analysed during this study are included in this published article (and its supplementary information files).

Scientific editing by Joel Smethurst; Fleur Loveridge

Appendix

Goodness-of-fit test, P *value*

The χ^2 test and Shapiro–Wilk goodness-of-fit test are used to determine whether sample data are consistent with a hypothesized probability density distribution. The hypothesis is as follows.

Null hypothesis (H_0): the data present a normal distribution

Alternative hypothesis (H_a): the data are not consistent with a normal distribution

In this case, a small *P* value (*P* value <0.05) corresponds to a null hypothesis rejection, which corresponds to the acceptance of the alternative hypothesis. Therefore, the data behave according to a not-normal probability density distribution.

Shapiro–Wilk. This test involves the correlation between the data and the corresponding normal scores, and tests the null hypothesis that a sample $x_1 \ldots x_n$ came from a normal probability density function. The test statistic is (e.g. Yazici and Yolacan 2007)

$$W = \frac{\left\{\sum_{i=1}^{k} a_{(n-i+1)}(x_{(n-i+1)} - x_{(i)}\right\}^2}{\sum_{i=1}^{n}(x_i - \bar{x})}$$

where x_i values are the ordered statistics.

Kolmogorov–Smirnov goodness-of-fit test. This is a numerical test of the empirical cumulative distribution function ($\tilde{F}$) against the fitted cumulative distribution ($\hat{F}$) (in this case a normal PDF). The Kolmogorov–Smirnov (KS) test seeks to see how close the empirical distribution is to the fitted cumulative distribution (Fenton and Griffiths 2008). The KS statistic D_n is defined as the largest vertical distance between $\tilde{F}_n(x)$ and $\hat{F}_n(x)$ across all values of x and is defined as

$$D_n = \max\{D_n^+, D_n^-\}$$

where

$$D_n^+ = \max_{1\le i\le n} \cdot \left\{\frac{i}{n} - \hat{F}_n(X_i)\right\}$$

$$D_n^- = \max_{1\le i\le n} \cdot \left\{\hat{F}_n(X_i) - \frac{i-1}{n}\right\}.$$

Linear regression

Correlations, laboratory results and the new q_u–N correlation were obtained through a linear regression analysis, which is represented as follows (e.g. Ang and Tang 2007):

$$\hat{y} = \beta_0 + \beta_1 x + e$$

where $\hat{y}$ is the regression dependent variable, x is the regression independent variable, β_0 and β_1 are regression coefficients obtained by the least-squares estimations; β_0 (regression intercept) is given as $\beta_0 = \bar{y} - \beta_1\bar{x}$, β_1 (regression slope) is given as $\beta_1 = \frac{\sum_{i=1}^{n} x_i y_i - n\overline{xy}}{\sum_{i=1}^{n} x_i^2 - n\bar{x}}$ where $\bar{x}$ and $\bar{y}$ are x and y mean values and n is the number of data, and e is the regression error $(y_i - \hat{y}_i)$ where y_i is the y value without the regression and $\hat{y}_i$ is the regression predicted value.

Correlation coefficient (R^2 *or* r^2*)*

This is a parameter that measures the percentage of the total variation of y that is explained by x, which is a scalar value that is between zero and unity (positive or negative depending on the regression slope). High R values indicate a good fit of the equation, and values close to zero suggest that x and y are not related. This value is obtained using the following equation (Weisberg 2005):

$$R^2 = \frac{\sum_{i=1}^{n}(\hat{y}_i - \bar{y})^2}{\sum_{i=1}^{n}(y_i - \bar{y})^2}.$$

Standard error of regression, s_e

The standard error of the estimate or regression is the estimated average distance that the observed values fall from the regression line. This value represents the regression dispersion around the mean such as:

$$s_e = \sqrt{\frac{\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}{n-2}}.$$

Analysis of variance (ANOVA), P *value*

ANOVA consists of calculations that provide information about the regression model levels of variability. ANOVA regression evaluation commonly implements the *P* value to test the null hypothesis. The *P* value provides another evaluation of the significance of the statistical relationship between the variables in the regression analyses. The hypothesis is as follows (Weisberg 2005):

Null hypothesis (H_0): $\hat{y} = \beta_0$

Alternative hypothesis (Ha): $\hat{y} = \beta_0 + \beta_1 x$

In this case, a small P value (P value <0.05) corresponds to a null hypothesis rejection, which corresponds to acceptance of the alternative hypothesis, giving validity to the regression model adopted.

Notation

PDF	probability density function
RBD	reliability-based design
SPT	standard penetration test
N value	number of blows obtained from SPT test
N_{field}	uncorrected number of blows obtained directly from the field
$(N_1)_{60}$	number of blows corrected for overburden and for 60% of energy
ϕ'	peak effective friction angle
C'	effective cohesion
q_u	undrained compressive strength
c_u	undrained shear strength
Cv	coefficient of variation
σ	standard deviation
μ	mean
P value	probability of a statistical hypothesis for a normality testing
R^2	coefficient of determination
σ'_n	effective normal stress
ϕ'_c	correlated effective friction angle

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